

RESEARCH ARTICLE

On totally $g^{\mu}b$ – Continuous functions in supra topological spaces

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Abstract

In this paper, we use $g^{\mu}b$ -closed set to define and investigate a new class of function namely totally $g^{\mu}b$ -continuous. Also compactness and convergence of totally $g^{\mu}b$ -continuous are discussed.

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Introduction

Functions and of course continuous functions stand among the most important and most researched points in the whole of the Mathematical Science. Many different forms of continuous functions have been introduced over the years. Some of them are totally continuous functions (Jain, 1980) strongly continuous functions (Levine, 1963), contra continuous functions (Dontchev, 1996). In 1980, Jain introduced totally continuous functions. Andrijevic (1996) obtained a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas (2004) under the name of γ -open sets. The notion of supra topological spaces was initiated by Mashhour *et al.* in 1983. In 2010, Sayed and Takashi Noiri introduced supra b-open sets and supra b-continuity on topological spaces. The purpose of this paper is to give some new type of continuity called totally $g^{\mu}b$ -continuity. Also we derived the properties of totally $g^{\mu}b$ -continuous and its compactness and convergence are also investigated.

1. Preliminaries

Definition: 1.1 (Mashhour *et al.*, 1983)

A subfamily μ of X is said to be a supra topology on X

if i) $X, \phi \in \mu$

ii) if $A_i \in \mu$ for all $i \in J$, then $\cup A_i \in \mu$. (X, μ) is called a supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of supra open set is called supra closed set and it is denoted by μ^c .

Definition: 1.2 (Mashhour *et al.*, 1983)

The supra closure of a set A is defined as $Cl^{\mu}(A) = \cap \{B: B \text{ is supra closed and } A \subseteq B\}$

The supra interior of a set A is defined as $Int^{\mu}(A) = \cup \{B: B \text{ is supra open and } A \supseteq B\}$

Definition: 1.3 (Arockiarani and Trinita Pricilla, 2011)

Let (X, μ) be a supra topological space. A set A of X is called supra generalized b-closed set (simply $g^{\mu}b$ -closed) if $bc1^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open. The complement of supra generalized b-closed set is supra generalized b-open set.

Definition: 1.4 (Sayed and Takashi Noiri, 2010)

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be $g^{\mu}b$ -continuous if $f^{-1}(V)$ is $g^{\mu}b$ -closed in (X, τ) for every supra closed set V of (Y, σ) .

Definition: 1.5 (Trinita Pricilla and Arockiarani, In Press)

A function $f: X \rightarrow Y$ is said to be $g^{\mu}b$ -totally continuous function if the inverse image of every $g^{\mu}b$ -open subset of Y is $Cl^{\mu}open^{\mu}$ in X .

Definition: 1.6 (Trinita Pricilla and Arockiarani, 2011)

A space (X, τ) is called T_{gb} -space if every $g^{\mu}b$ -closed set is b^{μ} -closed.

Definition: 1.7

A supra topological space X is said to be

- (i) Supra T_1 if for each pair of distinct points x and y of X , there exist supra opensets U and V containing x and y respectively such that $x \in U, y \notin U$ and $x \notin V, y \in V$.
- (ii) Supra T_2 if every two distinct points of X can be separated by disjoint supra open sets.

Definition: 1.8 (Trinita Pricilla and Arockiarani, In Press)
 A function $f: X \rightarrow Y$ is said to be supra-totally continuous function if the inverse image of every supra open subset of Y is $Cl^\mu open^\mu$ in X .

Definition: 2.9 (Trinita Pricilla and Arockiarani, In Press)
 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $g^\mu b$ -continuous if the inverse image of every $g^\mu b$ -open set of Y is supra open in (X, τ) .

2. Characterizations of totally $g^\mu b$ – continuous Functions

Definition: 2.1

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) totally $g^\mu b$ -continuous function if for each supra open subset V in Y containing $f(x)$, there exists a $g^\mu b - Cl^\mu open^\mu$ subset U in X containing x such that $f(U) \subset V$.
- (ii) totally $g^\mu b$ -continuous if it has the above property at each point of X .

Theorem: 2.2

The following statements are equivalent for a function $f: (X, \tau) \rightarrow (Y, \sigma)$:

- (i) f is totally $g^\mu b$ -continuous
- (ii) For every supra open set V of Y , $f^{-1}(V)$ is $g^\mu b - Cl^\mu open^\mu$ in X .

Proof: (i) \Rightarrow (ii) Let V be supra open subset of Y and let $x \in f^{-1}(V)$ be any arbitrary point. Since $f(x) \in V$ by (i), there exists $g^\mu b - Cl^\mu open^\mu$ set U_x in X containing x such that $U_x \subset f^{-1}(V)$. We obtain $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Since arbitrary union of $g^\mu b$ -open sets is $g^\mu b$ -open, $f^{-1}(V)$ is $g^\mu b - Cl^\mu open^\mu$ in X .

(ii) \Rightarrow (i) It is obvious.

Theorem: 2.3

- (i) Every Strongly $(g^\mu b)^*$ -continuous function is totally $g^\mu b$ -continuous.
- (ii) Every totally $g^\mu b$ -continuous function is $g^\mu b$ -continuous.
- (iii) Every $totally^\mu$ -continuous function is $g^\mu b$ -continuous.
- (iv) Every $totally^\mu$ -continuous function is totally $g^\mu b$ -continuous.
- (v) Every $g^\mu b$ -totally continuous function is $totally^\mu$ -continuous.
- (vi) Every $g^\mu b$ -totally continuous function is totally $g^\mu b$ -continuous.
- (vii) Every $g^\mu b$ -totally continuous function is $g^\mu b$ -continuous.
- (viii) Every $g^\mu b$ -totally continuous function is $strongly (g^\mu b)$ -continuous.

- (ix) Every Strongly $(g^\mu b)^*$ -continuous function is $strongly (g^\mu b)$ -continuous.

Proof: It is obvious.

Remark: 2.4

The converse of the above theorem is not true and it is shown by the following example.

Example: 2.5

Let $X = \{a, b, c, d\}; \tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{\phi, X, \{a\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be defined by $f(a) = b; f(b) = c; f(c) = d$ and $f(d) = a$. Here f is totally $g^\mu b$ -continuous function but not $totally^\mu$ continuous function. Also f is not $g^\mu b$ -totally continuous function.

Example: 2.6

Let $X = \{a, b, c\}; \tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be an identity function then f is $g^\mu b$ -continuous but $f^{-1}\{a\} = \{a\}$ is not $g^\mu b - Cl^\mu open^\mu$ in (X, τ) . Hence f is not totally $g^\mu b$ -continuous function.

Example: 2.7

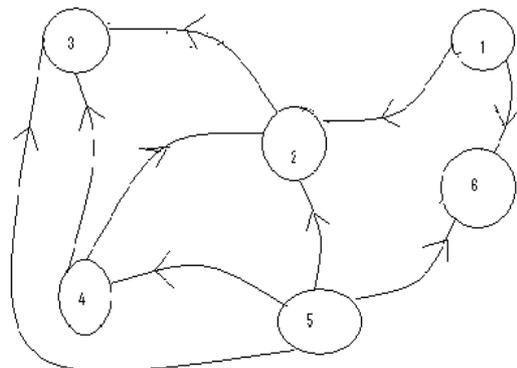
Let $X = \{a, b, c\}; \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Let $f: (X, \tau) \rightarrow (X, \tau)$ be an identity function then f is strongly $g^\mu b$ -continuous but $f^{-1}\{b\} = \{b\}$ is not $Cl^\mu open^\mu$ and $g^\mu b - Cl^\mu open^\mu$ in (X, τ) . Hence f is not $g^\mu b$ -totally continuous and Strongly $(g^\mu b)^*$ -continuous function.

Example: 2.8

Let $X = \{a, b, c, d\}; \tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{\phi, X, \{a\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be defined by $f(a) = b; f(b) = c; f(c) = d$ and $f(d) = a$. Here f is totally $g^\mu b$ -continuous function but $f^{-1}\{a\} = \{d\}$ is not $Cl^\mu open^\mu$ in (X, τ) . Hence f is not Strongly $(g^\mu b)^*$ -continuous function.

Remark: 2.9

From the above theorems and examples we have the following diagram:





In the above diagram, the numbers 1- 6 represent the following:

1. Strongly $(g^\mu b)^*$ -continuous function
2. totally $g^\mu b$ -continuous function
3. $g^\mu b$ -continuous function
4. $totally^\mu$ continuous function
5. $g^\mu b$ -totally continuous function
6. $strongly(g^\mu b)$ - continuous function

Definition: 2.10

A supra topological space (X, τ) is said to be $g^\mu b$ -connected if it is not the union of two non-empty disjoint $g^\mu b$ -open sets.

Theorem: 2.11

If f is totally $g^\mu b$ -continuous map from a $g^\mu b$ -connected space (X, τ) onto another space (Y, σ) , then (Y, σ) is an supra indiscrete space.

Proof: On the contrary suppose that (Y, σ) is not an supra indiscrete space. Let A be a proper non-empty supra open subset of (Y, σ) . Since f is totally $g^\mu b$ -continuous function, then $f^{-1}(A)$ is proper non-empty $g^\mu b - Cl^\mu open^\mu$ subset of X . Then $X = f^{-1}(A) \cup C(f^{-1}(A))$. Thus X is a union of two non-empty disjoint $g^\mu b$ -open sets which is a contradiction. Therefore Y must be an supra indiscrete space.

Theorem: 2.12

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be totally $g^\mu b$ - continuous function and Y is $g^\mu b$ - space. If A is non-empty $g^\mu b$ -connected subset of X , then $f(A)$ is singleton.

Proof: Suppose if possible $f(A)$ is not singleton. Let $f(x_1) = y_1 \in A$ and $f(x_2) = y_2 \in A$. Since $y_1, y_2 \in Y$ and Y is $g^\mu b$ - space, then there exists an $g^\mu b$ -open set G in (Y, σ) containing y_1 but not y_2 . Since f is totally $g^\mu b$ - continuous, then $f^{-1}(G)$ is $g^\mu b - Cl^\mu open^\mu$ set containing x_1 , but not x_2 . Now $X = f^{-1}(G) \cup C f^{-1}(G)$. Thus X is a union of two non empty $g^\mu b$ -open sets which is a contradiction.

Definition: 2.13

Let X be a supra topological space and $x \in X$. Then the set of all points y in X such that x and y cannot be separated by $g^\mu b$ -separation of X is said to be the quasi $g^\mu b$ -component of X .

Theorem: 2.14

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be totally $g^\mu b$ - continuous function from a supra topological space (X, τ) into a supra T_1 space Y . Then f is constant on each quasi $g^\mu b$ -component of X .

Proof: Let x and y be two points of X that lie in the same quasi $g^\mu b$ -component of X . Assume that $f(x) = \alpha \neq \beta = f(y)$. Since Y is supra T_1 , $\{\alpha\}$ is supra closed in Y and so $Y/\{\alpha\}$ is an supra open set. Since f is totally $g^\mu b$ - continuous, therefore $f^{-1}\{\alpha\}$ and $f^{-1}\{Y/\{\alpha\}$ are disjoint $g^\mu b - cl^\mu open^\mu$ subsets of X . Further, $x \in f^{-1}\{\alpha\}$ and $y \in f^{-1}\{Y/\{\alpha\}$ which is contradiction to the fact that y belongs to the quasi $g^\mu b$ -component of x and hence y must belong to every $g^\mu b$ -open set containing x .

Definition: 2.15

A space (X, τ) is said to be

- (i) $g^\mu b - co - T_1$ if for each pair of disjoint points x and y of X , there exists $g^\mu b-cl^\mu open^\mu$ sets U and V containing x and y , respectively such that $x \in U, y \notin U$ and $x \notin V, y \in V$.
- (ii) $g^\mu b - co - T_2$ if for each pair of disjoint points x and y of X , there exists $g^\mu b-cl^\mu open^\mu$ sets U and V in X , respectively such that $x \in U$ and $y \in V$.
- (iii) $g^\mu b - co - regular$ if for each $g^\mu b-cl^\mu open^\mu$ set F and each point $x \notin F$, there exists supra open sets U and V such that $F \subset U$ and $x \in V$.
- (iv) $g^\mu b - co - normal$ if for any pair of disjoint $g^\mu b-cl^\mu open^\mu$ subsets F_1 and F_2 of X , there exist disjoint supra open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem: 2.16

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is totally $g^\mu b$ - continuous injective function and Y is supra T_1 , then X is $g^\mu b - co - T_1$.

Proof: Suppose that Y is supra T_1 , for any distinct points x and y in X , there exist $V, W \in open^\mu(Y)$ such that $f(x) \in V, f(y) \notin V, f(x) \notin W$ and $f(y) \in W$. Since f is totally $g^\mu b$ - continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $g^\mu b-cl^\mu open^\mu$ subsets of (X, τ) such that $x \in f^{-1}(V), y \notin f^{-1}(V), x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is $g^\mu b - co - T_1$.

Theorem: 2.17

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is totally $g^\mu b$ - continuous injective function and Y is supra T_2 , then X is $g^\mu b - co - T_2$.

Proof: For any distinct points x and y in X , there exist disjoint supra open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is totally $g^\mu b$ - continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $g^\mu b-cl^\mu open^\mu$ in X containing x and y respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ because $U \cap V = \emptyset$. This shows that X is $g^\mu b - co - T_2$.

Theorem: 2.18

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is totally $g^\mu b$ - continuous injective supra open function from a $g^\mu b - co - normal$ Space X onto a space Y , then Y is supra normal.

Proof: Let F_1 and F_2 be disjoint supra open subsets of Y . Since f is totally $g^\mu b$ - continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are $g^\mu b-cl^\mu open^\mu$ sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. we have $U \cap V = \emptyset$. since X is $g^\mu b - co - normal$, there exist disjoint supra open sets A and B such that $U \subset A$ and $V \subset B$. we obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint supra open sets. Thus, Y is supra normal.

Theorem: 2.19

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is totally $g^\mu b$ - continuous injective supra open function from a $g^\mu b - co - regular$ Space X onto a space Y, then Y is supra regular.

Proof: It is similar to theorem 3.18.

Definition: 2.20

A supra topological space (X, τ) is said to be $g^\mu b - co - Hausdorff$ if every two distinct points of X can be separated by disjoint $g^\mu b - cl^\mu open^\mu$ sets.

Theorem: 2.21

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be totally $g^\mu b$ - continuous injection. If Y is supra hausdorff, then X is $g^\mu b - co - Hausdorff$.

Proof: Let x_1 and x_2 be two distinct points of X. Then since f is injective and Y is supra hausdorff, there exist $V_1, V_2 \in open^\mu(Y)$ such that $f(x_1) \in V_1, f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$.

By theorem 3.2, $x_i \in f^{-1}(V_i) \in g^\mu b - cl^\mu open^\mu(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, X is $g^\mu b - co - Hausdorff$.

Definition: 2.22

- (i) A filter base Λ is said to be *supra convergent* to a point x in X for any $U \in open^\mu(X)$ containing x , there exist $B \in \Lambda$ such that $B \subset U$.
- (ii) A filter base Λ is said to be $g^\mu b - co - convergent$ to a point x in X for any $U \in g^\mu bco(X)$ containing x , there exist $B \in \Lambda$ such that $B \subset U$.

Theorem: 2.23

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is totally $g^\mu b$ - continuous then for each point $x \in X$ and each filter base Λ in X $g^\mu b - co - converging$ to x , the filter base $f(\Lambda)$ is convergent to $f(x)$.

Proof: Let $x \in X$ and Λ be any filter base in X $g^\mu b - co - converging$ to x . Since f is totally $g^\mu b$ - continuous, then for any $V \in open^\mu(Y)$ containing $f(x)$, there exists a $U \in g^\mu b - cl^\mu open^\mu(X)$ containing x such that $f(U) \subset V$. Since Λ is $g^\mu b - co - converging$ to x , there exist $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Lambda)$ is convergent to $f(x)$.

Definition: 2.24

- (i) A space X is said to be $g^\mu b - co - compact$ if every $g^\mu b - cl^\mu open^\mu$ cover of X has a finite subcover.
- (ii) A space is said to be $g^\mu b - compact$ relative to X if every cover of a $g^\mu b - cl^\mu open^\mu$ sets of X has a finite subcover.
- (iii) A subset A of a space X is said to be $g^\mu b - compact$ if the subspace A is $g^\mu b - compact$.

Definition: 2.25

A space X is said to be

- (i) *Countably $g^\mu b - compact$* if every $g^\mu b - cl^\mu open^\mu$ countably cover of X has a finite subcover.
- (ii) $g^\mu b - co - Lindelof$ if every $g^\mu b - cl^\mu open^\mu$ cover of X has a countable subcover.
- (iii) $g^\mu b - closed compact$ if every $g^\mu b - cl^\mu open^\mu$ cover of X has a finite subcover.
- (iv) *Countably $g^\mu b - cl^\mu open^\mu - compact$* if every countably cover of X by $g^\mu b - cl^\mu open^\mu$ sets has a finite subcover.

Theorem: 2.26

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be totally $g^\mu b$ - continuous surjective function.

Then the following statements hold:

- (i) If X is $g^\mu b - co - Lindelof$ then Y is Lindelof
- (ii) If X is *Countably $g^\mu b - co - compact$* then Y is countably compact.

Proof: (i) Let $\{V_\alpha: \alpha \in I\}$ be an supra open cover of Y. Since f is totally $g^\mu b$ - continuous, then $\{f^{-1}(V_\alpha): \alpha \in I\}$ is $g^\mu b - cl^\mu open^\mu$ cover of X. Since X is $g^\mu b - co - Lindelof$, there exists a countable subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha): \alpha \in I_0\}$. Thus, $Y = \cup \{V_\alpha: \alpha \in I_0\}$ and hence Y is Lindelof.
 (ii) It is similar to (i)

Theorem: 2.27

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be totally $g^\mu b$ - continuous surjective function. Then the following statements hold:

- (i) If X is $g^\mu b - co - compact$, then Y is compact.
- (ii) If X is $g^\mu b - co - Lindelof$ then Y is Lindelof.
- (iii) If X is *Countably $g^\mu b - co - compact$* then Y is countably compact.

Proof: It is similar to theorem 3.22

Definition: 2.28

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $(g^\mu b)^*$ - continuous if the inverse image of every $g^\mu b$ -open set of Y is $Cl^\mu open^\mu$ in (X, τ) .

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